

Ground state heavy baryon production in a relativistic quark-diquark model

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We use current-current interaction to calculate the fragmentation functions to describe the production of spin-1/2, spin-1/2' and spin-3/2 baryons with massive constituents in a relativistic quark-diquark model. Our results are in their analytic forms and are applicable for singly, doubly and triply heavy baryons. We discuss the production of Ω_{bbc} , Ω_{bcc} and Ω_{ccc} baryons in some detail. The results are satisfactorily compared with those obtained for triply heavy baryons calculated in a perturbative regime within reasonable values of the parameters involved.

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I. INTRODUCTION

Heavy baryon physics is one of the important topics in recent particle physics. One of the reasons is the improvement of the experimental data [1]. Another reason rests on the theoretical side where valuable phenomenological aspects of treating such states have been improved. Quantum chromodynamics serves as a leading candidate to study the production and decay properties of such states. Heavy quark effective theory [2] has also proved to be a useful tool in situations where heavy quark and light degree(s) of freedom both are present, namely, for singly and doubly heavy baryons. It has proved to be successful in some circumstances specially where the heavy quark mass could be approximated infinity with respect to mass scale in the process [3]. Heavy baryons have also been studied in relativistic quark models mainly in favor of singly and doubly heavy baryons [4]. Another valuable idea is the application of diquarks [5]. Undoubtedly it reduces the task of treating a three body system to a two body system. Its application in the case of light quarks have proved to be successful in most of the situations.

Description of hadronic properties is accomplished by the set of Dyson-Schwinger equations [6]. The simplest hadrons are mesons which are color singlet bound states of a quark and an anti-quark. They are described by solutions of the homogenous Bethe-Salpeter equation (BSE) for $\bar{q}q$ states. The Bethe-Salpeter amplitudes of different types of mesons such as pseudo-scalar, vector, etc. are characterized by different Dirac structures. In addition to $\bar{q}q$ bound states, there are qq states by studying the corresponding BSE. Single gluon exchange leads to an interaction that is attractive for diquarks in a color antitriplet configuration. Two quarks can be coupled in either a color sextet or a color antitriplet state. Furthermore, it is the diquark in a color antitriplet state that

can couple with a quark to form a color-singlet baryon. Therefore only the antitriplet configuration of qq states are considered here. Similar to the case of mesons, the different types of diquarks are characterized by different Dirac structures.

On the other hand we have an interesting situation regarding heavy baryons in which different combinations of light and heavy flavor give raise to singly, doubly and triply heavy states. Wide variety of speculations have been applied to understand the production and decay properties of these states. It seems that it would be interesting to consider a case in which all possibilities are involved benefiting the idea of diquark. In combining a quark with a diquark, the state of the diquark will play an important role. While the combination of a scalar diquark with a heavy quark will end up with a spin-1/2 baryon, the similar situation with a vector diquark will produce either a spin-1/2 (generally called as spin-1/2') or a spin-3/2 baryon. Such a scenario is to be studied in more detail here where the constituents are assumed massive and relativistic. We will focus our attention to the production of such states, and obtain three fragmentation functions to describe the fragmentation production of them and compare our results with available theoretical results.

Our strategy is as follows. We discuss the heavy flavor diquarks in section II. In section III we specify the spin wave functions for baryons in their ground and in possible spin states. The section IV is devoted to obtain the fragmentation functions and we illustrate the application of our results in the case of Ω_{bbc} , Ω_{bcc} and Ω_{ccc} baryons in section V where we also discuss our results.

II. HEAVY FLAVOR DIQUARKS

Application of diquark model is achieved by calculating the Feynman diagrams with rules for point-like particles. To embed the composite nature of a diquark, phenomenological vertex functions should be introduced. Parameterization of the 3-point functions and diquark form

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factors is obtained from the requirement that, asymptotically the diquark model evolves into the pure quark model. Moreover that there is no direct information about chromomagnetic form factors. We may expect that the ordinary electromagnetic form factors will have the same functional form as their QCD counterparts since the source of both form factors is the matrix elements of a conserved vector operator. Here the vector operator is the color octet gluon field. The parameterization of the diquark form factors may be inferred from quark-diquark models of the nucleons. In this view the relevant form factors in the space-like region are written as [7]

$$\begin{aligned} F_S(Q^2) &= \left(1 + \frac{Q^2}{Q_S^2}\right)^{-1}, & F_E(Q^2) &= \left(1 + \frac{Q^2}{Q_V^2}\right)^{-2}, \\ F_M(Q^2) &= (1 + \kappa)F_E(Q^2), & F_Q(Q^2) &= 0. \end{aligned} \quad (1)$$

It is assumed that the scalar and vector diquark form factors to have simple and dipole forms respectively with pole positions at Q_S and Q_V above 1 GeV [8]. In the above, κ is the anomalous chromomagnetic dipole moment. The form factor related to chromoelectric quadrupole moment is set equal to zero [9]. In the case of heavy diquarks, the following expressions for the above form factors are also proposed

$$\begin{aligned} F_S(Q^2) &= \frac{Q_S^2}{Q^2}, & F_E(Q^2) &= \left(\frac{Q_V^2}{Q^2}\right)^2, \\ F_M(Q^2) &= (1 + \kappa)F_E(Q^2), & F_Q(Q^2) &= 0. \end{aligned} \quad (2)$$

The above form factors are more consistent where heavy flavor is involved in heavy baryon production. The parameters Q_S and Q_V are free parameters and have a crucial role in determination of the fragmentation probabilities. We will comment more about the values of Q_S and Q_V in the last section.

The gluon-diquark coupling will be different for scalar and vector diquarks. The form of these couplings are determined by the four-momentum flow and polarization four vectors in the Feynman diagram [10]. The Feynman rule for the coupling of a gluon to a scalar diquark (SgS-vertex) reads as

$$ig_s T_{ij}^a (k + k'), \quad (3)$$

and for similar coupling for a vector diquark (VgV-vertex), we have

$$\begin{aligned} ig_s T_{ij}^a \{ &g_{\alpha\beta}(k + k')_\mu - g_{\mu\alpha}[(1 + \kappa)k - \kappa k']_\beta \\ &- g_{\mu\beta}[(1 + \kappa)k' - \kappa k]_\alpha \}. \end{aligned} \quad (4)$$

Here k and k' are four momenta for diquark and anti-diquark respectively.

III. GROUND STATE SPIN WAVE FUNCTIONS FOR HEAVY DIQUARKS AND HEAVY BARYONS

In this section we try to investigate the way in which a heavy quark and a heavy diquark combine together to form a heavy baryon. It is assumed that the diquark and the produced baryon are in their ground states. We emphasize that while the ground state diquark composed of different flavors is a scalar spin-0 state, a diquark with identical constituents should be considered in a vector spin-1 state due to Fermi statistics.

In the quark-diquark model, ground state heavy baryons are composed of a heavy quark Q and a heavy diquark system D with spin-0 or spin-1, moving in a S-wave state. We denote the spin wave functions of a heavy spin-0 diquark by χ and a spin-1 diquark by χ^μ . These functions are normalized according to [3]

$$(\chi, \chi) = 1, \quad (\chi^\mu, \chi^\nu) = -g^{\mu\nu} + \frac{k^\mu k^\nu}{m_D^2}. \quad (5)$$

It is sometimes convenient to transform to the spherical basis for the spin-1 diquark which can be done with the help of the spin-1 polarization vector. For $\lambda = \pm 1, 0$, we may write

$$\chi(1, \lambda) = \epsilon_\mu(\lambda) \chi^\mu, \quad (6)$$

and the inverse

$$\chi^\mu = \sum_\lambda \epsilon^{*\mu}(\lambda) \chi(1, \lambda), \quad (7)$$

where the polarization four-vector for different helicity states are defined as usual

$$\begin{aligned} \epsilon_\mu(\pm) &= \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0), \\ \epsilon_\mu(0) &= (|\mathbf{k}|, 0, 0, k_0). \end{aligned} \quad (8)$$

Since the spin wave functions χ and χ^μ satisfy the Bargmann-Wigner equation on both spinor labels, they are spin wave functions built from constituent on-mass shell quarks [3].

In combining a heavy quark and a diquark, one obtains a ground state heavy baryon. The ground state diquark is either in spin-0 or in spin-1 state. Therefore there will be different possibilities for the spin state of the heavy baryon. These possibilities are shown in Table I. The spin wave function $\Psi_{\alpha\beta\gamma}$ of a ground state heavy baryon is written down by invariant coupling between the diquark spin tensors χ and χ^μ and the heavy quark spinor tensors u and u^μ . The heavy quark spinor tensors u and u^μ involve the heavy baryon spinor U_B and the Rarita-Schwinger spinor vector Ψ^μ . We then may write [3]

$$\begin{aligned}
&\text{Scalar diquark; spin } -1/2 \text{ baryon : } \Psi_{\alpha\beta\gamma} = \chi_{\alpha\beta} u_\gamma \equiv \chi U_B, \\
&\text{Vector diquark; spin } -1/2' \text{ baryon : } \Psi_{\alpha\beta\gamma} = \chi_{\alpha\beta}^\mu u_{\mu,\gamma} \equiv \chi^\mu \left\{ \frac{1}{\sqrt{3}} \gamma^\mu \gamma_5 U_B \right\}, \\
&\text{Vector diquark; spin } -3/2 \text{ baryon : } \Psi_{\alpha\beta\gamma} = \chi_{\alpha\beta}^\mu u_{\mu,\gamma} \equiv \chi^\mu \Psi_\mu.
\end{aligned} \tag{9}$$

TABLE I: The ground state triply heavy baryons with corresponding possible spin states when considered to be formed in quark-diquark model. The relevant diquark spin states are also shown.

Baryon Formation Process	Diquark spin	Baryon spin
$c \rightarrow \Omega_{ccc}, \Omega_{ccc}^*$	1	$\frac{1}{2}, \frac{3}{2}$
$b \rightarrow \Omega_{bbb}, \Omega_{bbb}^*$	1	$\frac{1}{2}, \frac{3}{2}$
$b \rightarrow \Omega_{bbc}$	0	$\frac{1}{2}, \frac{1}{2}$
$c \rightarrow \Omega_{bbc}, \Omega_{bbc}^*$	1	$\frac{1}{2}, \frac{3}{2}$
$c \rightarrow \Omega_{bcc}$	0	$\frac{1}{2}, \frac{1}{2}$
$b \rightarrow \Omega_{bcc}, \Omega_{bcc}^*$	1	$\frac{1}{2}, \frac{3}{2}$

In the constituent quark model, the explicit forms of a scalar and a vector diquark spin wave functions are respectively given by [11]

$$\begin{aligned}
\chi &= \frac{1}{2\sqrt{2}m_D} [(\not{k} + m_D)\gamma_5], \\
\chi^\mu &= \frac{1}{2\sqrt{2}m_D} [(\not{k} + m_D)\gamma^\mu].
\end{aligned} \tag{10}$$

Therefore the spin wave functions for the heavy baryons may be constructed from these ingredients.

IV. FRAGMENTATION FUNCTIONS

The formation of a baryon in a quark-diquark model is described in Fig. 1 where the diquark which attaches to the initial state heavy quark may be in a scalar or a vector state. Fragmentation is usually described by the function $D(z, \mu_o)$, where z being the energy-momentum fraction taken by the baryon and μ_o is the scale at which such function is calculable in perturbative QCD. This function may be put in the following form

$$D_{Q \rightarrow QQ'Q''}(z, \mu_o) = \frac{1}{2} \sum_s \int d^3\mathbf{p} d^3\mathbf{k} d^3\mathbf{k}' |T_B|^2 \times \delta^3(\mathbf{p} + \mathbf{k} + \mathbf{k}' - \mathbf{p}'), \tag{11}$$

where T_B is the scattering amplitude for the fragmentation process. The average over initial spin state and summation over all final state particle spins are included for production of unpolarized baryon state.

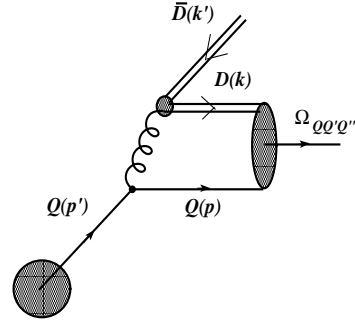


FIG. 1: The lowest order Feynman diagram contributing to the fragmentation of a heavy quark (Q) into a heavy baryon with three massive constituents ($\Omega_{QQ'Q''}$) in a quark-diquark model. The four momenta are labelled. The diquark D consists of two heavy flavor Q' and Q'' and \bar{D} of \bar{Q}' and \bar{Q}'' .

To obtain the hard scattering amplitude, we consider the currents produced by the diquark pair and the initial state heavy quark with appropriate coupling to a vector gluon. Such a current for a scalar diquark may be put in the following form

$$j_S^\mu \sim g_s F_S(Q^2) q^\mu \bar{\chi}'(k') \cdot \chi(k) e^{-iq \cdot x}, \tag{12}$$

where k and k' are the scalar diquark and anti-diquark four-momenta with $q = k + k'$ and $F_S(Q^2)$ is the relevant form factor given in (1). Here $Q^2 = -q^2$. The tensors $\chi(k)$ and $\chi'(k')$ are the spin wave functions for diquark and anti-diquark respectively. Similarly the current for a vector diquark with a coupling to a vector gluon casts into the following [12]

$$j_V^\mu \sim g_s \left\{ -F_E(Q^2) [\chi(k) \cdot \bar{\chi}'(k')] q^\mu + F_M(Q^2) [(k \cdot \chi'(k')) \bar{\chi}^\mu + (k' \cdot \bar{\chi}(k)) \chi'^\mu] \right\} e^{-iq \cdot x}. \tag{13}$$

Here $F_E(Q^2)$ and $F_M(Q^2)$ are chromoelectric and chromomagnetic form factors given in (1). The current produced by the initial state heavy quark with a coupling to a vector gluon may be written in the following form

$$j_\mu^Q \sim g_s [\bar{u}(p) \gamma_\mu u(p')] e^{-i(p-p') \cdot x}. \quad (14)$$

Now we write down the general form of the amplitudes for production of a ground state heavy baryon with possible spin states. In doing so first we form the hard scattering amplitude using the currents (12), (13) and (14). Then we convolute the result with the momentum space distribution amplitude to obtain the total amplitude. The distribution amplitude is chosen to have the following form

$$\Phi_B = \frac{f_B}{m_B} \delta\left(x_i - \frac{m_i}{m_B}\right), \quad (15)$$

where m_B and f_B are mass and decay constant for the

baryon. With the distribution amplitude given by (15), it is assumed that constituents of the baryon move almost collinearly and that the effects of Fermi motion is disregarded. Eventually in the first order perturbation theory the total amplitude for the process in discussion, in a general case, will have the following form

$$T_{S,V} = -i \int [dx] d^4x \Phi_B \left\{ j_{S,V}^\mu \left(\frac{1}{q^2} \right) j_\mu^Q \right\}, \quad (16)$$

where $[dx] = dx_1 dx_2 \delta(1 - x_1 - x_2)$, x_1 and x_2 being the energy momentum ratios carried by the heavy quark and the diquark. The currents $j_{S,V}^\mu$ and j_μ^Q are given by (12), (13) and (14). We are now in a position to write down the amplitudes corresponding to the possible attachments of the initial state heavy quark with the diquark to form a ground state baryon with a definite spin state. For a scalar diquark to combine with a heavy quark giving rise to a spin-1/2 triply heavy baryon, we have

$$T_{S1/2} = \frac{g_s^2 C_F f_B}{8m_B^2 \sqrt{2p_0 p'_0 k_0 k'_0}} \frac{F_S(Q^2)}{D_\circ q^2} \{ \bar{U}_B [\chi'(P + m_B) \gamma_5 (\not{k}' + m_D)] u(p') \}. \quad (17)$$

In the above, the conservation of three momentum is implicit. Since the initial state heavy quark is not on its mass shell, we have energy non-conservation, and have performed the energy integration to reproduce the energy denominator D_\circ in (17), where $D_\circ = p_\circ + k_\circ + k'_\circ - p'_\circ$. q is the momentum transferred by the gluon. We have followed our arguments in section III to calculate the Dirac

structure in (17).

When the diquark is vector, there are two configurations for addition of spin angular momenta and moreover that for each case there are contributions from chromoelectric and chromomagnetic form factors. For production of the so called spin-1/2' baryon with chromoelectric form factor we have

$$T_{VE1/2'} = \frac{g_s^2 C_F f_B}{2m_B^2 \sqrt{6p_0 p'_0 k_0 k'_0}} \frac{F_E(Q^2)}{D_\circ q^2} \{ \bar{U}_B [\chi'(P + m_B) \gamma_5 (\not{k}' + m_D)] u(p') \}. \quad (18)$$

Similar the amplitude with chromomagnetic form factor is put in the following form

$$T_{VM1/2'} = \frac{g_s^2 C_F f_B}{2m_B^2 \sqrt{6p_0 p'_0 k_0 k'_0}} \frac{F_M(Q^2)}{D_\circ q^2} \{ \bar{U}_B [2\beta(P, \chi') \gamma_5 (\not{P} - 2m_B) + \not{k}' \gamma_5 (\not{P} + m_B) \not{\chi}'] u(p') \}. \quad (19)$$

Likewise there are two contributions for spin 3/2 baryon formation. They are

$$T_{VE3/2} = -\frac{g_s^2 C_F f_B}{8m_B^2 \sqrt{2p_0 p'_0 k_0 k'_0}} \frac{F_E(Q^2)}{D_\circ q^2} \{ \bar{\Psi}_{3/2}^\mu [\not{\chi}' \gamma_\mu (\not{P} + m_B) (\not{k}' + m_D)] u(p') \}, \quad (20)$$

and

$$T_{VM3/2} = \frac{g_s^2 C_F f_B}{8m_B^2 \sqrt{2p_0 p'_0 k_0 k'_0}} \frac{F_M(Q^2)}{D_\circ q^2} \{ \bar{\Psi}_{3/2}^\mu [2\beta(P, \chi') (2\not{P}_\mu - m_B \gamma_\mu) + \not{k}' \gamma_\mu (\not{P} + m_B) \not{\chi}'] u(p') \}. \quad (21)$$

For a spin-3/2 baryon with four momentum P , summation over helicity states is carried out by means of the following projection operator [13]

$$\mathcal{P}_{3/2}^{\mu\nu}(P) = g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{1}{3P^2}(P\gamma^\mu P^\nu + P^\mu\gamma^\nu P). \quad (22)$$

Next we calculate the fragmentation functions. For spin-1/2 baryon including a scalar diquark we put (17) into (11). Performing average/sum over initial/final spin states we find

$$D_{1/2}(z, \mu_o) = \frac{1}{2} \left(\frac{\pi\alpha_s f_B C_F}{m_B} \right)^2 \int \frac{d^3\mathbf{p} d^3\mathbf{k} d^3\mathbf{k}' \delta^3(\mathbf{p} + \mathbf{k} + \mathbf{k}' - \mathbf{p}') F_S^2(Q^2)}{D_o^2 q^4 p_0 p'_0 k_0 k'_0} \times \{ (p'.k')(P.k') + \beta(p'.k') + \alpha\beta(P.k') + \alpha\beta^2 \}. \quad (23)$$

In the case of spin-1/2' baryon, employing a vector diquark, we should add up the chromoelectric and chromomagnetic contributions given by (18) and (19). In this way we conclude that

$$D_{1/2'}(z, \mu_o) = \frac{2}{3} \left(\frac{2\pi\alpha_s f_B C_F}{m_D m_B^2} \right)^2 \int \frac{d^3\mathbf{p} d^3\mathbf{k} d^3\mathbf{k}' \delta^3(\mathbf{p} + \mathbf{k} + \mathbf{k}' - \mathbf{p}') F_E^2(Q^2)}{D_o^2 q^4 p_0 p'_0 k_0 k'_0} \times \kappa(P.k')^2 \{ \kappa[(p'.k')(P.k') + \beta(p'.k') + \beta^2(P.p') + \alpha\beta(P.k')] + \beta^2(P.p') - \alpha\beta^2 \}. \quad (24)$$

Similarly for a spin-3/2 baryon we obtain

$$D_{3/2}(z, \mu_o) = \frac{(\pi\alpha_s f_B C_F)^2}{3m_D m_B^5} \int \frac{d^3\mathbf{p} d^3\mathbf{k} d^3\mathbf{k}' \delta^3(\mathbf{p} + \mathbf{k} + \mathbf{k}' - \mathbf{p}') F_E^2(Q^2)}{D_o^2 q^4 p_0 p'_0 k_0 k'_0} \times \{ 4\kappa(1+\kappa)(p'.k')(P.k')^2 + \beta(3\kappa^2 + 5\kappa - 1)(p'.k')(P.k') - \beta^2(2-\kappa)(p'.k') + 2\kappa(1+\kappa)(P.p')(p.k')^3 + \beta(3\kappa^2 + \kappa - 2)(P.p')(P.k')^2 - \beta^2(\kappa + 4)(P.p')(p.k') - 3\beta^3(P.p') + 6\alpha\kappa(1+\kappa)(P.k')^3 + 3\alpha\beta(2\kappa^2 + 2\kappa - 1)(P.k')^2 - 6\alpha\beta^2(P.k') - 3\alpha\beta^3 \}. \quad (25)$$

We have set up our kinematics such that the dot products in (23), (24) and (25), take the following form

$$\begin{aligned} 2P.k'/m_B^2 &= \frac{z}{1-z}(\beta^2 + \gamma^2) + \frac{1-z}{z}, & 2P.p'/m_B^2 &= z(\alpha^2 + \gamma^2) + 1/z, \\ 2p'.k'/m_B^2 &= \frac{1}{1-z}(\beta^2 + \gamma^2) + (1-z)(\alpha^2 + \gamma^2) - 2\gamma^2, \end{aligned} \quad (26)$$

where we have defined

$$\alpha = \frac{m_Q}{m_B}, \quad \beta = \frac{m_D}{m_B}, \quad \gamma = \frac{k'_T}{m_B}. \quad (27)$$

Finally to obtain the explicit form of the fragmentation functions, we perform the phase space integrations [14]. In this way we find the fragmentation functions for spin-1/2, 1/2' and 3/2 baryons as follows. For spin-1/2 case we have

$$D_{1/2}(z, \mu_o) = \frac{\pi^3}{2} \left(\frac{\alpha_s C_F f_B Q_S^2}{m_B^3} \right)^2 F_1(z), \quad (28)$$

where $F_1(z)$ is given by

$$\begin{aligned} F_1(z) &= \{ z^4(1-z)^4[\beta^2 - 2\alpha\beta(-1+z) + \alpha^2(1-z)^2 + \gamma^2 z^2] \} / \\ &\quad \{ [1 - (1 - 2\alpha\beta - 2\beta^2)z - (2\alpha\beta + \beta^2 - \gamma^2)z^2]^2 \\ &\quad \times [1 + 2(-1 + \beta)z + (1 - 2\beta + \beta^2 + \gamma^2)z^2]^3 \}. \end{aligned} \quad (29)$$

Here we have redefined $\gamma = \langle k'_T \rangle^2 / m_B$. The fragmentation function for spin-1/2' case is obtained as

$$D_{1/2'}(z, \mu_o) = \frac{\pi^3}{3} \left(\frac{4\alpha_s C_F f_B Q_V^4}{\beta^2 m_B^5} \right)^2 \frac{F_2(z)}{G(z)}. \quad (30)$$

Here the functions $F_2(z)$ and $G(z)$ are given by

$$F_2(z) = \kappa f_1(z) + \kappa^2 f_2(z), \quad (31)$$

and

$$G(z) = [1 - (1 - 2\alpha\beta - 2\beta^2)z - (2\alpha\beta + \beta^2 - \gamma^2)z^2]^2 \times [1 - 2(1 - \beta)z + (1 - 2\beta + \beta^2 + \gamma^2)z^2]^6, \quad (32)$$

where

$$f_1(z) = 2\beta^2 z^4 (1 - z)^6 (1 - 2\alpha z + \alpha^2 z^2 + \gamma^2 z^2) [1 - 2z + (1 + \beta^2 + \gamma^2)z^2]^2, \quad (33)$$

$$\begin{aligned} f_2(z) = & z^4 (1 - z)^4 [1 - 2z + (1 + \beta^2 + \gamma^2)z^2]^2 \\ & \times \{ \beta^4 z^2 + 2\beta^3 z(1 - z)(1 + \alpha z) + [\alpha^2 (1 - z)^2 + \gamma^2 z^2] [1 - 2z + (1 + \gamma^2)z^2] \\ & + \beta^2 [3 - 6z + 3(1 + \alpha^2 + \gamma^2)z^2 - 2(3\alpha^2 + 2\gamma^2)z^3 + 3(\alpha^2 + \gamma^2)z^4] \\ & + 2\beta(1 - z)[\alpha^2 z(1 - z)^2 + \gamma^2 z^3 + \alpha(1 - 2z + (1 + \gamma^2)z^2)] \}. \end{aligned} \quad (34)$$

Finally for a spin-3/2 baryon we have

$$D_{3/2}(z, \mu_o) = \frac{\pi^3}{6\beta^3} \left(\frac{\alpha_s C_F f_B Q_V^4}{m_B^5} \right)^2 \frac{F_3(z)}{G(z)}. \quad (35)$$

Here the function $F_3(z)$ reads as

$$F_3(z) = f'_1(z) + \kappa f'_2(z) + \kappa^2 f'_3(z), \quad (36)$$

where

$$\begin{aligned} f'_1(z) = & -2\beta z^4 (1 - z)^6 [1 - 2z + (1 + \beta^2 + \gamma^2)z^2] \\ & \times \{ 1 - (2 - 3\alpha - 4\beta)z + [1 + 2\alpha^2 - 4\beta + 2\beta^2 - 6\alpha(1 - 2\beta) + 2\gamma^2]z^2 \\ & - [4\alpha^2(1 - \beta) + 2(1 - 2\beta)\gamma^2 - 3\alpha(1 - 4\beta + \beta^2 + \gamma^2)]z^3 + (\alpha^2 + \gamma^2)(2 - 4\beta + \beta^2 + \gamma^2)z^4 \}, \end{aligned} \quad (37)$$

$$\begin{aligned} f'_2(z) = & z^4 (1 - z)^4 [1 - 2z + (1 + \beta^2 + \gamma^2)z^2] \\ & \times \{ \beta^3 z^2 (1 - z)^2 (11 + \gamma^2 z^2) + \beta^4 z^2 [2 - 2(1 - \gamma^2)z^2 + \gamma^2 z^4] \\ & + \beta(1 - z)^2 [1 - 2z + (1 + 2\gamma^2)z^2 - 2\gamma^2 z^3 + \gamma^2(11 + \gamma^2)z^4] \\ & + \gamma^2 z^2 [1 - 2(3 - \gamma^2)z^2 + 8z^3 - (3 + 2\gamma^2)z^4] + 6\alpha(1 - z)[1 - 2(2 - \beta)z \\ & + 2(3 - 3\beta + \beta^2 + \gamma^2)z^2 - 2(2\beta^2 - \beta^3 + 2(1 + \gamma^2) - \beta(3 + \gamma^2))z^3 \\ & - (2\beta(1 + \gamma^2) - 2\beta^2(1 + \gamma^2))z^4] + \alpha^2(1 - z)^2 [1 - (6 - 11\beta - 2\beta^2 - 2\gamma^2)z^2 + 2(4 - 11\beta - \beta^2)z^3 \\ & - (3 + 2\gamma^2 - \beta(11 + \gamma^2))z^4] + \beta^2 [1 - 2z + 2\gamma^2 z^2 + 2(1 - \gamma^2)z^3 - (1 - 6\gamma^2)z^4] \}, \end{aligned} \quad (38)$$

and

$$\begin{aligned} f'_3(z) = & z^4 (1 - z)^4 [1 - 2z + (1 + \beta^2 + \gamma^2)z^2] \\ & \times \{ 3\beta^3 z^2 (1 - z)^2 (3 + \gamma^2 z^2) + \beta^4 z^2 [2 - 2(1 - \gamma^2)z^2 + \gamma^2 z^4] \\ & + 3\beta(1 - z)^2 [1 - 2z + (1 + 2\gamma^2)z^2 - 2\gamma^2 z^3 + \gamma^2(3 + \gamma^2)z^4] \\ & + \gamma^2 z^2 [1 - 2(3 - \gamma^2)z^2 + 8z^3 - (3 + 2\gamma^2)z^4] + 6\alpha(1 - z)[1 - 2(2 - \beta)z + 2(3 - 3\beta + \beta^2 + \gamma^2)z^2 \\ & - 2(2\beta^2 + 2(1 + \gamma^2) - \beta(3 + \gamma^2))z^3 + (-2\beta(1 + \gamma^2) + 2\beta^2(1 + \gamma^2))z^4] \\ & + \alpha^2(1 - z)^2 [1 + (-6 + 9\beta + 2\beta^2 + 2\gamma^2)z^2 + (8 - 18\beta)z^3 - (3 + 2\gamma^2 + 2\beta^2(1 - \gamma^2) - 3\beta(3 + \gamma^2))z^4] \\ & + \beta^2(1 - 2(3 - \gamma^2)z^2 + 8z^3 - 3z^4 - 2\gamma^2(1 - \gamma^2)z^6) \}. \end{aligned} \quad (39)$$

form factors given in (2). We have explained this in more detail in the following section.

V. RESULTS AND DISCUSSION

The fragmentation functions given by (28), (30) and (35) are obtained assuming three massive constituents for the final state baryon in relativistic manner. They may be applied to different baryonic states. Note that we have only considered the diquarks and baryons in their ground states. While the fragmentation function (28) provides the description of a spin-1/2 baryon with a scalar diquark, (30) and (35) will demonstrate the production of spin-1/2' and spin-3/2 baryons with a vector diquark. For singly heavy baryons one should consider a light diquark to attach to an initial state heavy quark. For doubly heavy baryons the attachment of a light-heavy diquark to a heavy quark should be considered. In this case the diquark in its ground state is a scalar diquark. Therefore only the fragmentation function (28) may be used to describe spin-1/2 doubly heavy baryons.

As a typical application of this model, we consider the situation of triply heavy baryons and as a prototype example for application of (28), we consider the fragmentation production of Ω_{bbc} and Ω_{bcc} in a b or a c quark fragmentation. Here the diquark is a colored bc state. Using this function we have obtained the behavior of the fragmentation functions for these states. The behavior of the fragmentation functions are shown in Fig. 2. The fragmentation probabilities as well as the average fragmentation parameters are also calculated using (28). These results are in fair agreement with similar results ones in [16]. An interesting and useful example for application of (30) and (35) is the situation of fragmentation production of Ω_{ccc} baryon. According to our early discussion, the vector cc diquark will produce either a spin-1/2', Ω_{ccc} , or a spin-3/2, Ω_{ccc}^* , state in a c quark fragmentation. Figure (3) describes the situation. Although the quantity $\langle z \rangle$ is a little bit higher for the spin-3/2' case, nevertheless the fragmentation probabilities are almost the same. This would mean that the cross sections for Ω_{ccc} and Ω_{ccc}^* would be the same at any colliding facility at least within the framework of our study. This result is mainly because we have ignored the Fermi motion inside the baryon and m_B has been taken the same for both Ω_{ccc} and Ω_{ccc}^* . These results are comparable with those obtained in [14] and [15] where triply heavy baryons are studied in detail using perturbation theory and without the idea of diquarks. Table II shows all physical observables which results from (30) and (35) for these two states.

For the fragmentation functions obtained in (28), (30) and (35) we have used the input parameters as those employed in [14], whose results are compared with those obtained here. The heavy quark masses are $m_b = 4.25$ GeV and $m_c = 1.25$ GeV. For all triply heavy baryons

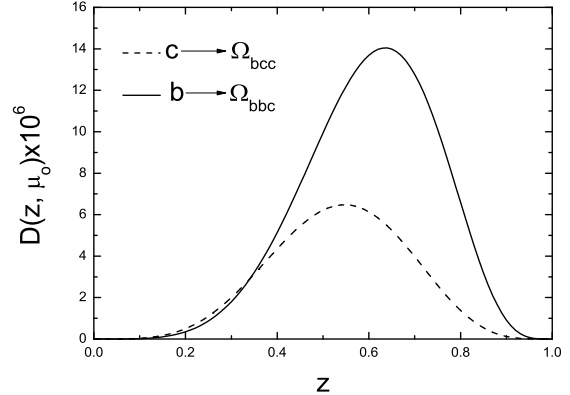


FIG. 2: The fragmentation of c and b quarks into Ω_{bcc} and Ω_{bbc} baryons respectively. Here the diquark is a scalar bc . The curves are plotted using (28).

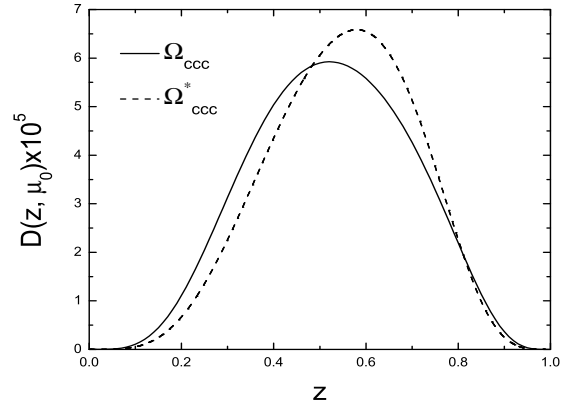


FIG. 3: Fragmentation of a c quark into a spin-1/2, Ω_{ccc} or a spin-3/2, Ω_{ccc}^* baryon as obtained from (30) and (35) respectively. Here the diquark is a vector cc .

we have chosen the decay constant to be $f_B = 0.25$ GeV. The color factor for the color structure of the diagram in Fig. 1 is $C_F = 4/(3\sqrt{3})$. We have run the coupling constant α_s according to appropriate momentum flow.

Application of the diquark form factors has introduced the parameters Q_S and Q_V into our description for which there is not much information at hand. Perturbative calculations of [14] provide a clue. Although the fragmentation functions in the two models are different, we assume that the fragmentation probabilities provided by them for a given state are equal. This procedure will fix the value of corresponding Q_S (Q_V). The fragmentation functions for production of Ω_{ccc} and Ω_{ccc}^* is shown in Fig. 4 from which we obtain the value of Q_V for Ω_{ccc} and Ω_{ccc}^* production. Similarly the study of other processes will determine the corresponding Q_S (Q_V) for them. Some of

TABLE II: The fragmentation probabilities (F.P.), the average fragmentation parameter $\langle z \rangle$ and the cross sections obtained from (28), (30) and (35) for production of the states specified. The values of the parameters Q_S and Q_V are also given. Total cross sections are calculated at the scale of $\mu = 2\mu_R$ [14].

Process	$\langle z \rangle$	F.P.	$Q_S(Q_V)[\text{GeV}]$	Cross section (LHC) [pb]
$b \rightarrow \Omega_{bbc}$	0.603	5.12×10^{-6}	7.0	35
$c \rightarrow \Omega_{bcc}$	0.535	2.47×10^{-6}	4.9	30
$c \rightarrow \Omega_{ccc}$	0.526	2.77×10^{-5}	2.9	310
$c \rightarrow \Omega_{ccc}^*$	0.549	2.76×10^{-5}	2.9	310

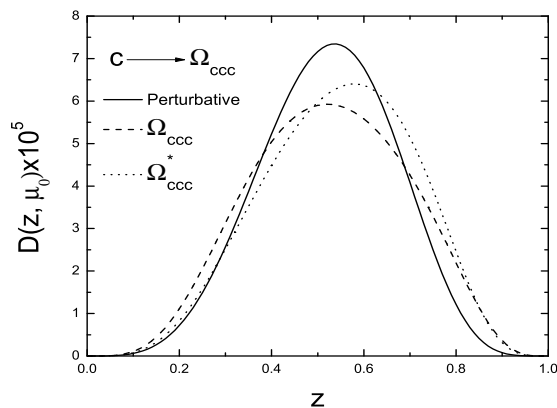


FIG. 4: Obtaining C_V for the process $c \rightarrow \Omega_{ccc}$ or Ω_{ccc}^* , from comparison of the two models, the perturbative approach and quark-diquark models. The fragmentation probabilities are taken to be the equal.

the values for these parameters are given in Table II. It is interesting to note that the values of these parameters increase as the mass of the baryon increases by inclusion of heavier and heavier flavors. The next input parameter is the anomalous chromomagnetic dipole moment κ . Different values have been attributed by different authors for this quantity. Our study shows that among them the value of $\kappa = 1.39$ is more consistent with the behavior of the fragmentation functions and physical quantities extracted from them.

At the end we would like to add that the ordinary parameterization for the form factors describing the coupling of a gluon to heavy diquark, i.e. the forms introduced in (1), just fail to provide reasonable behavior of the fragmentation functions, one needs abnormally large values of Q_S and Q_V to bring about reasonable fragmentation probabilities. However the forms in (2) look justified.

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